Shell effects for small Ising clusters

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A key to an analysis of nuclear multifragmentation data leading to the nuclear matter phase diagram [1] was Fisher's droplet model [2]. At coexistence Fisher's model gives the temperature T cluster yields as

$$n_s(T) \propto g(s)\exp(-ws/T) \qquad (1)$$

where s is the cluster's surface area, $g(s)$ is proportional to the cluster's degeneracy, w is the surface tension.

Based on the combinatorics of two dimensional clusters Fisher suggested $g(s)$ would be given by

$$g(s) \propto s^{-\tau/\sigma}\exp(\zeta s) \qquad (2)$$

where τ and σ are critical exponents and is the surface entropy tension. Over a limited range in temperature he mean surface area of a cluster of A constituents can be approximated as

$$\langle s\rangle = a_0 A^{\sigma} \qquad (3)$$

where a_0 is a geometric prefactor. Putting Eqn's (1), (2) and (3) together gives

$$n_A(T) \propto A^{-\tau}\exp(w a_0 A^{\sigma}\varepsilon/T). \qquad (4)$$

where $\varepsilon = 1 - T\zeta/w = 1 - T/T_c$ is a measure of the distance from the critical temperature.

For sufficiently large clusters on a two dimensional square lattice $a_0 \sim 4$ and $\sigma \sim 1/2$. However, for small cluster shell effects will play a major role. For example, monomers have $s = 4$, dimers have $s = 6$ and trimers have $s = 8$. Clusters with $A = 4$ can have either $s = 8$ or $s = 10$.

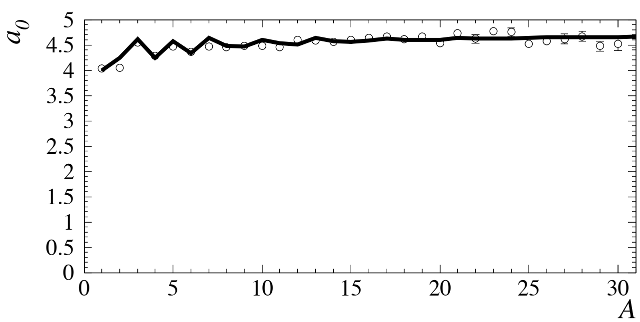

FIG. 1: Geometric prefactor as a function of cluster number.

To study the shell effects of a_0 cluster yields from two dimensional square Ising lattices were used for $n_A(T)$ and Eq. (4) was solved for $a_0(A)$ using the standard two dimensional Ising values of $\tau = 31/15$, $\sigma = 8/15$, $T_c = 2.26915$ and $w = 2$. The results are shown by the open circles in Fig. 1 where the shell effects are seen clearly for clusters with $A \leq 10$ before the limiting behavior of $a_0 \approx 4.6$ sets in.

The same shell effects for small clusters are evident when cluster yields are generated using Eq. (1) and the counted degeneracy of $g(s,A)$ [3] giving

$$n_A(T) \propto \sum_s g(s,A)\exp(ws/T). \qquad (5)$$

Examples of the values of $g(s,A)$ are given in Fig. 2. The geometrical prefactor is then

$$a_0 = A^{-\sigma}\langle s\rangle = A^{-\sigma}\frac{\sum_s s g(s,A)\exp(ws/T)}{\sum_s g(s,A)\exp(ws/T)}. \qquad (6)$$

The solid line in Fig. 1 gives the result for Eq. (6) evaluated at $T = 1$ approximately half of T_c. Again the shell effects are seen clearly for clusters with $A \leq 10$ before the limiting behavior of $a_0 \approx 4.6$ sets in.

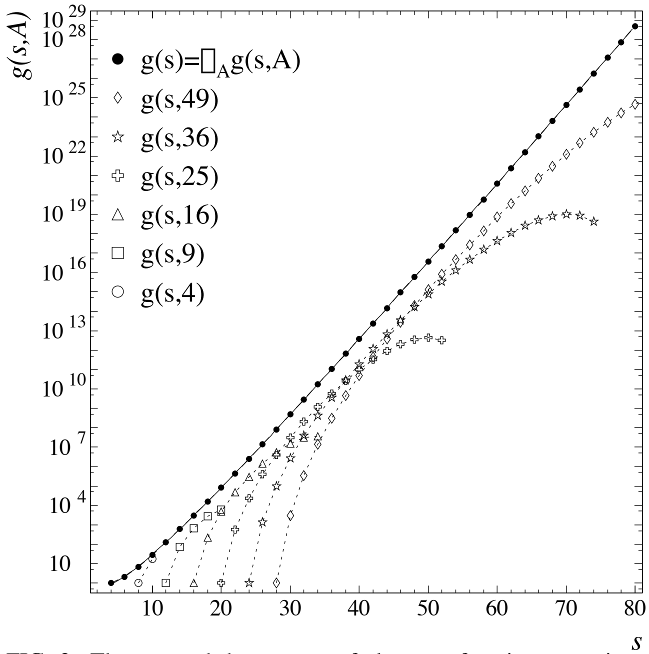

FIG. 2: The counted degeneracy of clusters of a given constituent number and surface.

These results show the approximation of Eq. (3) is good to $\sim 1\%$ for $A > 10$ but good only to $\sim 10\%$ for $A \leq 10$.

[1] J. B. Elliott *et al.*, Phys. Rev. C **67**, 024609 (2003) .
[2] M. E. Fisher, Physics **3**, 255 (1967).
[3] I. Jensen, private communication (2003).